Development of a Mathematical Model for Particle-Droplet Interaction for Dust Control

• Introduction
• Problem description
• Mathematical basis for model
• Development of model
• Discussion of preliminary results
• Conclusions / Future Work
Introduction

• Significance of dust control for sustainable development
  – The health dangers with respect to air quality have been known for over 500 years.
  – Protecting health of mine workers
  – Protecting health/quality of life of surrounding community
  – Improving reputation of mining company
Dust Fractions

EUROSIL
Dust – Mining

• The major health concerns with regards to respirable dust in mining are:
  – Coal workers’ pneumoconiosis
  – Silicosis

• Dust originates from:
  – Development
  – Extraction
  – Conveying and Hauling
Water Sprays Systems for Dust Control
Basics of Particle-Droplet Interaction

- Dust particles
  - Particle size distribution
  - Concentration
- Water droplets
  - Droplet size distribution
  - Flow rate
- Interaction
  - Relative velocity

- Assumptions
  - Dust particles entrained in uniform air flow
  - Water droplet stationary
  - Low Reynolds Number / Stokes Flow
  - Drag Force is critical
Mathematical Basis for Model

- **Streamlines**

\[
\psi = Ur^2 \sin^2 \theta \left( \frac{1}{2} - \frac{3a}{4r} + \frac{a^3}{4r^3} \right)
\]

\[
u_r = \frac{1}{r^2 \sin \theta \partial_r} = U \cos \theta \left( 1 - \frac{3a}{2r} + \frac{a^3}{2r^3} \right)
\]

\[
u_\theta = -\frac{1}{r \sin \theta \partial_r} = -U \sin \theta \left( 1 - \frac{3a}{4r} - \frac{a^3}{4r^3} \right)
\]

- **Drag Force**

\[
F_D = C_D A \frac{\rho u^2}{2}
\]

\[
C_D = \frac{24}{Re}
\]

\[
Re = \frac{\rho u D}{\mu}
\]

\[
F_D = 6\pi \mu R_p u
\]
### Development of Model: Parameters

<table>
<thead>
<tr>
<th>Factor</th>
<th>Variable</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dust Particle Size</td>
<td>$R_p$ (radius)</td>
<td>0.25 – 5 µm</td>
</tr>
<tr>
<td>Dust particle density</td>
<td>$D_p$</td>
<td>1 – 3 t/m³</td>
</tr>
<tr>
<td>Water droplet size</td>
<td>$R_d$ (radius)</td>
<td>5 – 50 µm</td>
</tr>
<tr>
<td>Uniform of fluid</td>
<td>$U$</td>
<td>1 – 6 m/s</td>
</tr>
<tr>
<td>Mass of particle</td>
<td>$m$</td>
<td>$D_p \times (4\pi R_p^3/3)$</td>
</tr>
<tr>
<td>Velocity of fluid at point</td>
<td>$u_f$</td>
<td>vector</td>
</tr>
<tr>
<td>Starting distance of particle</td>
<td>$h$</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Time</td>
<td>$t$</td>
<td>0 – 0.02 s</td>
</tr>
<tr>
<td>Velocity of particle</td>
<td>$u$</td>
<td>vector</td>
</tr>
<tr>
<td>Density of fluid</td>
<td>$d_f$</td>
<td>1.184 kg/m³</td>
</tr>
<tr>
<td>Kinematic viscosity of fluid</td>
<td>$Nu$</td>
<td>$1.562 \times 10^{-3}$ m²/s</td>
</tr>
</tbody>
</table>
Development of Model: Physics

• Newton’s second law for the particle gives

the sum of forces acting on the particle equals the mass of the particle times its acceleration

or

\[ F_D = ma \]
Development of Model: Initial Value Problem

• Initial value problem (IVP) for the velocity of the dust particle $u(t)$:

$$m \, u' = 6\pi \, R_p \, (u_f - u) \, , \text{ with } u \text{ as time dependent velocity of particle;}$$

$$u(0) = u_f$$
Dependencies in the IVP

- Note that the fluid velocity vector $\mathbf{u}_f = \langle u_r, u_\theta \rangle$ changes implicitly and non-linearly with respect to time through its dependencies on $r = r(t)$ and $\theta = \theta(t)$:

\[
\begin{align*}
    u_r &= \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} = U \cos \theta \left(1 - \frac{3a}{2r} + \frac{a^3}{2r^3}\right) \\
    u_\theta &= -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r} = -U \sin \theta \left(1 - \frac{3a}{4r} - \frac{a^3}{4r^3}\right)
\end{align*}
\]
Approach to Solving the IVP

• The IVP reduces to a differential equation for the velocity of the particle,

\[ m \mathbf{u}' = 6\pi R_p (\mathbf{u}_f (r(t), \theta(t)) - \mathbf{u}) \]

together with the initial condition,

\[ \mathbf{u}(0) = \mathbf{u}_f \]

• The implicit dependence on time of the differential equation requires numerical computation of \( \mathbf{u} \)…
Algorithm for Solving the IVP

- Numerical Method for the solution to the IVP: Runge Kutta of order 2 (RK2)
- Algorithm (implemented in Matlab):
  a. Given initial r and θ, compute fluid velocity \( u_f(r, \theta) \);
  b. Compute new particle velocity \( u \) from IVP using RK2;
  c. Compute new position of particle based on new velocity \( u \);
  d. Compute new \((r, \theta)\) based on the new position;
  e. Repeat steps (a-d) until particle collides with the droplet.
Particles Colliding with Droplet

Particles from different starting heights, hitting the droplet

- Droplet
- Particle Path
Discussion of preliminary results

- Variation of 4 primary variables
- Calculation of maximum initial height to ensure collision
- Plot of maximum initial height vs. variable
- Determine dependencies
Max. initial height with respect to particle size

The graph shows the initial height of a particle vs the radius of the particle. The conditions are: fluid velocity is 6, density of the particle is 2500, and the radius of the droplet is 50 x 10^-6.
Max. initial height with respect to density
Max. initial height with respect to droplet size
Max. initial height with respect to fluid velocity
Future Work

• Use of higher order Runge-Kutta method to improve accuracy
• Improve accuracy of drag force calculation with Oseen’s approximation
• Find a mathematical description of all parameters with respect to particle-droplet collisions
• Use information regarding dust concentration and water flow rates to determine the dust capture efficiency of a water spray system.
• Make a user friendly interface
Conclusion

• Successfully completed first step in modeling dust particle and water droplet interaction for dust control
• Model behaves as logically expected
• Next steps can be implemented
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