A Geostatistical Study of Lignite Spatial Variations in the Multilayer Deposit of the Amyndeo Mine in Northern Greece

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The Amyndeo mine

- Area 17 km² (6,6 sq. miles) with an extensive fault system. The lignite deposit is multilayered.
- PPC has produced 107 Mt since 1989
- Average annual Lignite production 8 Mt
- Estimated mean calorific value around 1200 kcal/kg
Satellite image (Google Earth)
Stages of Geostatistical study

- Exploratory statistical analysis
- Trend modelling
- Variogram estimation
- Reserves estimation
- Kriging (Spatial interpolation and mapping)
2D plot of drill hole locations
Exploratory analysis

- Drill hole data from 2 areas of the mine.

- The data used are the total thickness of lignite layers at the drill hole locations.

- Evaluation criteria: lignite layers with ash and CO$_2$ content over 50% or thickness less than 0.5 m are discarded.
# Table of statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; dataset (exhausted)</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; dataset (total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>302</td>
<td>376</td>
</tr>
<tr>
<td>Mean (m)</td>
<td>13.26</td>
<td>14.67</td>
</tr>
<tr>
<td>Median (m)</td>
<td>12.85</td>
<td>14.10</td>
</tr>
<tr>
<td>Standard Dev (m)</td>
<td>7.60</td>
<td>8.44</td>
</tr>
<tr>
<td>Range (m)</td>
<td>36.60</td>
<td>48.00</td>
</tr>
<tr>
<td>Minimum (m)</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Maximum (m)</td>
<td>37.10</td>
<td>48.50</td>
</tr>
</tbody>
</table>
Trend Model

\[ m_{Ze}(s) = 15.13 - 4.07x + 2.56y - 0.13x^2 - 1.96y^2 - 1.93xy \]
\[ m_{Zt}(s) = 18.61 - 4.95x + 2.31y - 1.97x^2 - 2.17y^2 - 1.22xy \]

- Correlation coefficient of trend with 1st dataset
  \[ \rho_{z,me} = 67.4\% \]

- Correlation coefficient of trend with 2nd dataset
  \[ \rho_{z,mt} = 65.2\% \]

- The fluctuations \( X_i = Z_i - m_z \), do not follow the normal distribution in either datasets (based on the Lilliefors test at the 10% significance level).
Lignite thickness trend
Variogram estimation for the exhausted part of the mine

- The robust estimator (Cressie and Hawkins, 1980) of the experimental variogram is used to reduce impact of extreme values.

- The semivariogram plots for the exhausted area suggest geometric anisotropy.

- The method of maximum likelihood is used to fit the variograms to an anisotropic exponential model.

<table>
<thead>
<tr>
<th>Anisotropic ratio</th>
<th>$\varphi$</th>
<th>$\xi_x$</th>
<th>$C_0$ (m)</th>
<th>$\xi_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.90</td>
<td>99.4°</td>
<td>0.210</td>
<td>2.65</td>
<td>0.399</td>
</tr>
</tbody>
</table>

SDIMI 2007 MILOS
Variogram plots for the exhausted part of the mine

Variogram directions: $\pi/3$, $2\pi/3$, $\pi$. Tolerance: $\pi/4$
Variogram estimation for the whole mine

- The semivariogram plots for the total mine area do not suggest geometric anisotropy.
- From the directional semivariograms, the anisotropy ratio is estimated to be from 1.45 to 1.7.
- The omnidirectional experimental variogram is calculated.
Variogram plots for the total mine area
Variogram models

- **Exponential model:**

\[
\gamma (r) = c_0 + \sigma^2 \left[ 1 - \exp \left( -\frac{\|r\|}{\xi} \right) \right]
\]

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\sigma^2$ (m)</th>
<th>$C_0$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.26</td>
<td>17.97</td>
<td>14.17</td>
</tr>
</tbody>
</table>

- **Spartan model:**

\[
\gamma (r) = \frac{\eta_0 \xi^2}{2\pi} \int_0^{\kappa_c} dk \frac{k \left[ 1 - J_0(kr) \right]}{1 + \eta_1 \xi^2 k^2 + \xi^4 k^4}
\]

<table>
<thead>
<tr>
<th>$\eta_0$</th>
<th>$\eta_1$</th>
<th>$\xi$</th>
<th>$k_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>514.92</td>
<td>2.26</td>
<td>0.06</td>
<td>32.13</td>
</tr>
</tbody>
</table>
Maps of estimated lignite thickness and estimate precision are generated with Ordinary Kriging using the exponential variogram.

The map is generated on a 100 x 100 grid. Each grid cell is 50.8m x 34.8m.

Correlation coefficient between actual and estimated values is 68.4%.

Estimation of reserves using this grid is 205 Mt.
Estimated lignite thickness (m)
Kriging standard deviation (m)
Reserves Estimation

- Voronoi diagrams are used to estimate mine reserves.
- Each point of the study area is considered to have the total thickness of the lignite layers of the nearest drill hole.
- A group of mirror points is used to close the Voronoi polygons at the boundaries of the study area.
Reserves estimation in excavated area

Estimation: 125 Mt
PPC data: 105 Mt

Overestimation is about 20%.
Total reserves estimation

- **Initial Estimation:** 287 Mt
- **Assumed overestimation:** 55 - 60 Mt
- **Final estimation:** 230 Mt
Conclusions

- The Voronoi diagrams give a reasonable first estimate of the lignite reserves even with a restricted set of drill hole evaluation criteria.

- The OK precision is not satisfactory. Partly because of the non-Gaussian distribution of the lignite thickness, partly because of the faults of the area.
Suggestions for further analysis

- Incorporation of a more complete set of drill hole evaluation criteria.
- Modelling non-Gaussian thickness fluctuations.
- Application of the Spartan Random Fields model for Ordinary Kriging (using a denser drill hole grid that will be available in the near future).
- Creation of Kriging estimation maps on other lignite properties like flying ash content and calorific value.
- Using constrained Voronoi diagrams to better account for the presence of the faults.
Thank you!

Acknowledgments

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(EPEAEK-II) PYTHAGORAS
Robust estimator (Cressie and Hawkins, 1980)

\[ \gamma_{\text{exp}}(\mathbf{r}) = \left[ \sum_{i=1}^{N_r} \sqrt{ \frac{\left| Z(s_i) - Z(s_i + \mathbf{r}) \right|}{N_r} } \right]^4 \]

\[ = \frac{\left[ \sum_{i=1}^{N_r} \sqrt{ \frac{\left| Z(s_i) - Z(s_i + \mathbf{r}) \right|}{N_r} } \right]^4}{2 \left( \frac{0.494}{N_r} + 0.457 \right)} \]
Variance of Ordinary Kriging estimation

\[ \sigma^2_{OK}(s_u) = \sum_{i=1}^{4} \lambda_i \gamma(|| s_u - s_i ||) + \mu \]

\[ \sum_{i=1}^{4} \lambda_i = 1 \]