# Oregon TECH

# Development of a Mathematical Model for Particle-Droplet Interaction for Dust Control

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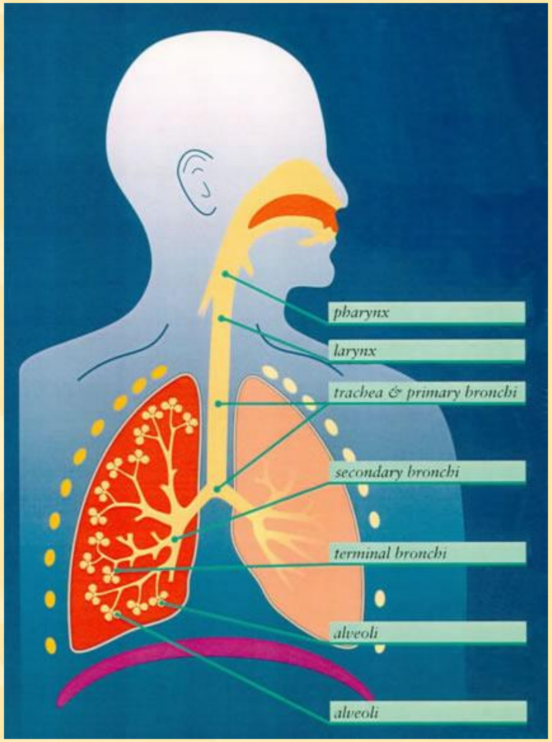
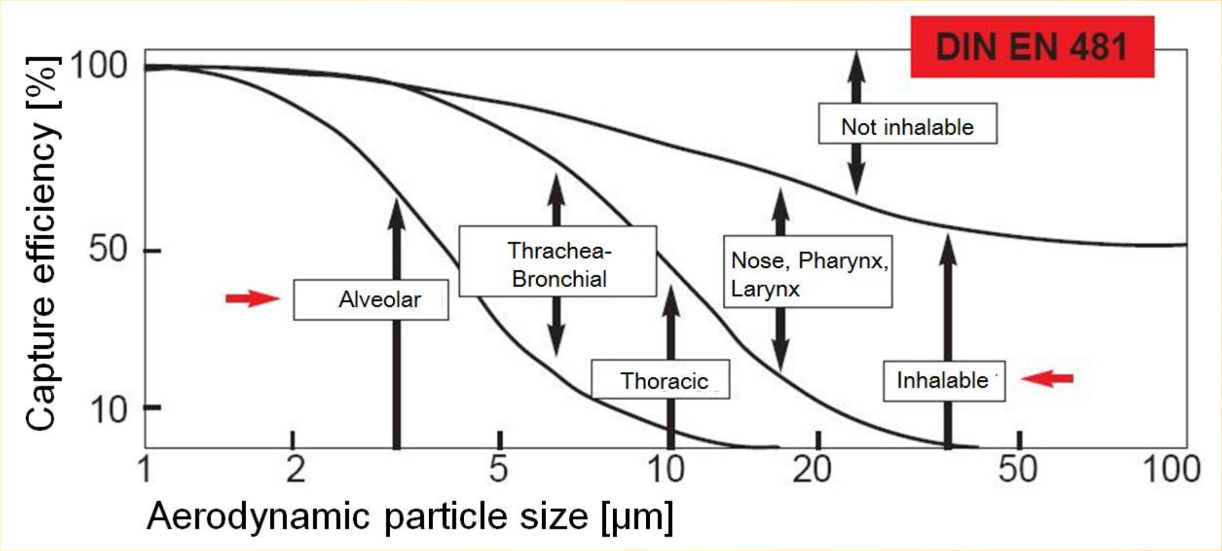
- Introduction
- Problem description
- Mathematical basis for model
- Development of model
- Discussion of preliminary results
- Conclusions / Future Work

# Introduction

- Significance of dust control for sustainable development
  - The health dangers with respect to air quality have been known for over 500 years.
  - Protecting health of mine workers
  - Protecting health/quality of life of surrounding community
  - Improving reputation of mining company

# Dust Fractions

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# Dust – Mining

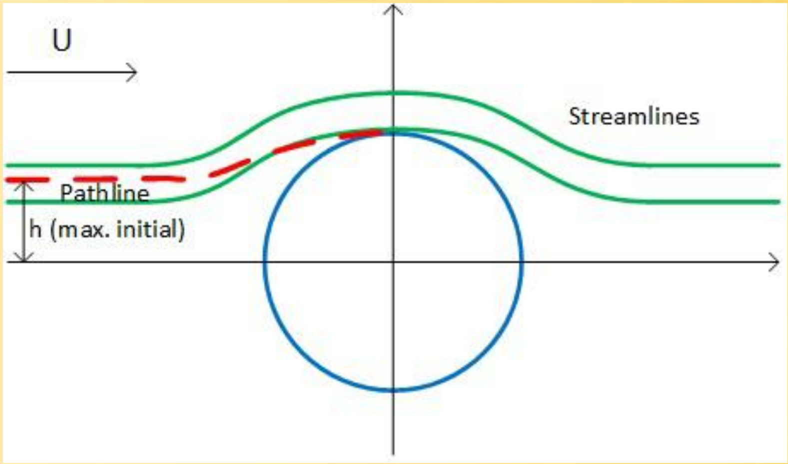
- The major health concerns with regards to respirable dust in mining are:
  - Coal workers' pneumoconiosis
  - Silicosis
- Dust originates from:
  - Development
  - Extraction
  - Conveying and Hauling

# Water Sprays Systems for Dust Control

# Basics of Particle-Droplet Interaction

- Dust particles
  - Particle size distribution
  - Concentration
- Water droplets
  - Droplet size distribution
  - Flow rate
- Interaction
  - Relative velocity

- Assumptions
  - Dust particles entrained in uniform air flow
  - Water droplet stationary
  - Low Reynolds Number / Stokes Flow
  - Drag Force is critical



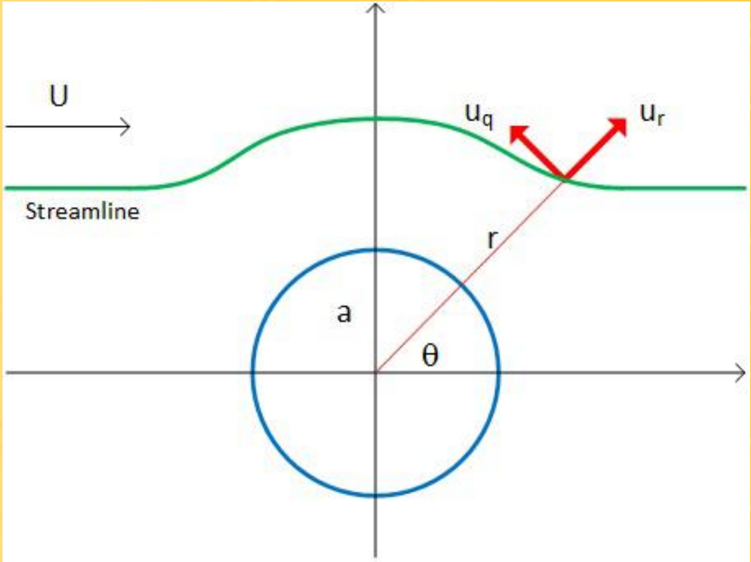
# Mathematical Basis for Model

- Streamlines

$$\psi = Ur^2 \sin^2\theta \left\{\frac{1}{2} - \frac{3a}{4r} + \frac{a^3}{4r^3}\right\}$$

$$u_r = \frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta} = U\cos\theta\left\{1 - \frac{3a}{2r} + \frac{a^3}{2r^3}\right\}$$

$$u_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} = -U\sin\theta\left\{1 - \frac{3a}{4r} - \frac{a^3}{4r^3}\right\}$$




- Drag Force

$$F_D = C_D A\frac{\rho u^2}{2} \quad C_D = \frac{24}{Re} \quad Re = \frac{\rho u D}{\mu} \Rightarrow F_D = 6\pi\mu R_p u$$

# Development of Model: Parameters

| **Factor** | **Variable** | **Range** |
|---|---|---|
| Dust Particle Size | $R_p$ (radius) | 0.25 – 5 µm |
| Dust particle density | $D_p$ | 1 – 3 t/m$^3$ |
| Water droplet size | $R_d$ (radius) | 5 – 50 µm |
| Uniform  of fluid | U | 1 – 6 m/s |
| Mass of particle | m | $D_p * (4\pi R_p^3/3)$ |
| Velocity of fluid at point | $\mathbf{u_f}$ | vector |
| Starting distance of particle | h | 0.1 m |
| Time | t | 0 – 0.02 s |
| Velocity of particle | $\mathbf{u}$ | vector |
| Density of fluid | $d_f$ | 1.184 kg/m$^3$ |
| Kinematic viscosity of fluid | Nu | 1.562 x 10$^{-3}$ m$^2$/s |

# Development of Model: Physics

- Newton’s second law for the particle gives

*the sum of forces acting on the particle equals the mass of the particle times its acceleration*

or

$$F_D = ma$$

# Development of Model: Initial Value Problem

- Initial value problem (IVP) for the velocity of the dust particle $\mathbf{u}(t)$:

$m\mathbf{u}' = 6\pi R_p (\mathbf{u_f} - \mathbf{u})$ , with $\mathbf{u}$ as time dependent velocity of particle;

$\mathbf{u}(0) = \mathbf{u_f}$

# Dependencies in the IVP

- Note that the fluid velocity vector $\mathbf{u_f}$ = <$u_r$, $u_\theta$> changes implicitly and *nonlinearly* with respect to *time* through its dependencies on r = r(t) and θ = θ(t):

$$u_r = \frac{1}{r^2 \sin\theta} \frac{\partial \psi}{\partial \theta} = U \cos\theta \left\{1 - \frac{3a}{2r} + \frac{a^3}{2r^3}\right\}$$

$$u_\theta = -\frac{1}{r \sin\theta} \frac{\partial \psi}{\partial r} = -U \sin\theta \left\{1 - \frac{3a}{4r} - \frac{a^3}{4r^3}\right\}$$

# Approach to Solving the IVP

- The IVP reduces to a differential equation for the velocity of the particle,

$$m\mathbf{u}' = 6\pi R_p (\mathbf{u_f} (r(t), \theta(t)) - \mathbf{u})$$

together with the initial condition,

$$\mathbf{u}(0) = \mathbf{u_f}$$

- The implicit dependence on time of the differential equation requires numerical computation of **u**…

# Algorithm for Solving the IVP

- Numerical Method for the solution to the IVP: Runge Kutta of order 2 (RK2)
- Algorithm (implemented in Matlab):

  a. *Given initial r and θ, compute fluid velocity* $\boldsymbol{u_f}$ *(r, θ);*

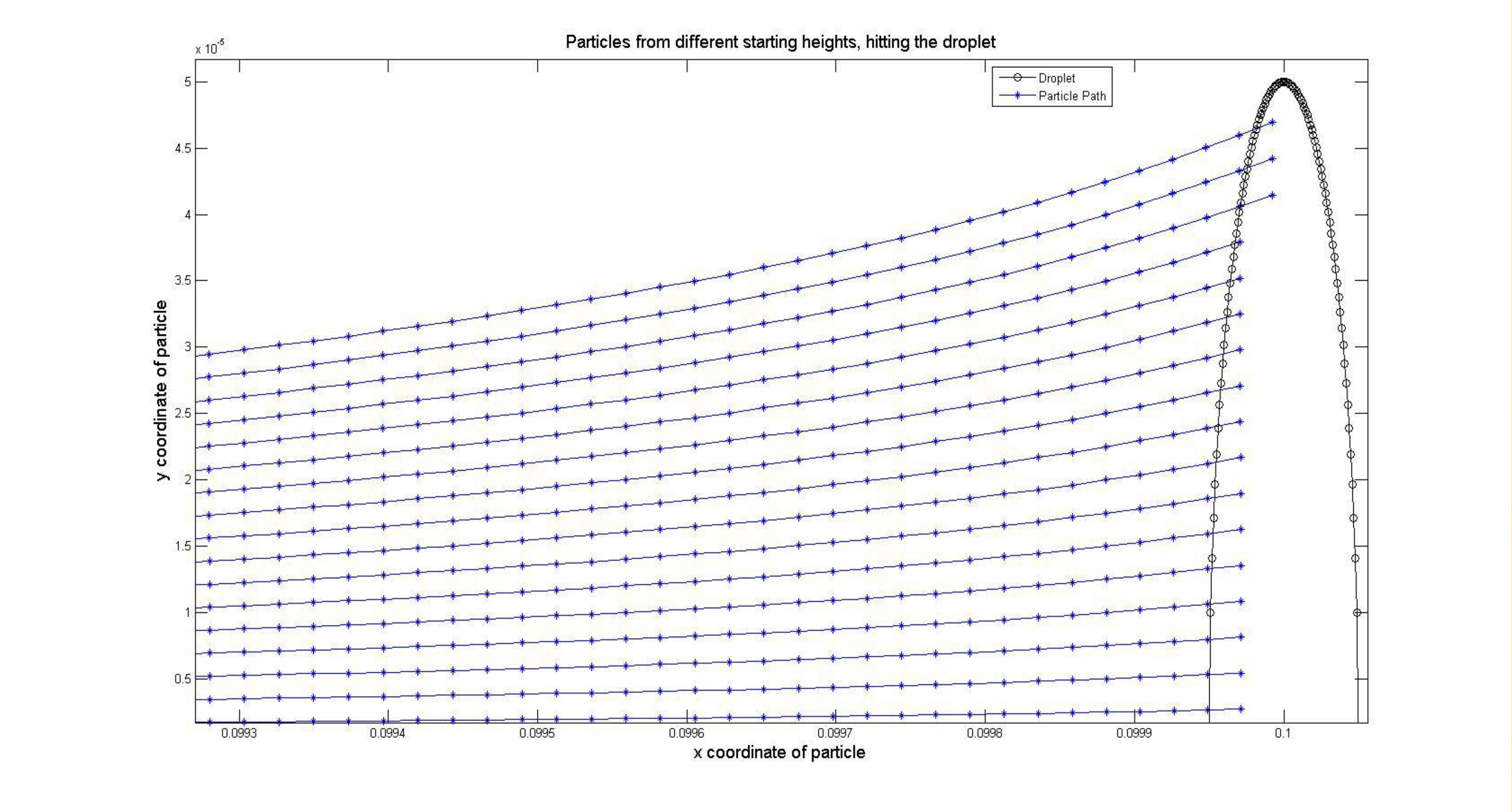
  b. *Compute new particle velocity* $\boldsymbol{u}$ *from IVP using RK2;*

  c. *Compute new position of particle based on new velocity* $\boldsymbol{u}$*;*

  d. *Compute new (r, θ) based on the new position;*

  e. *Repeat steps (a-d) until particle collides with the droplet.*

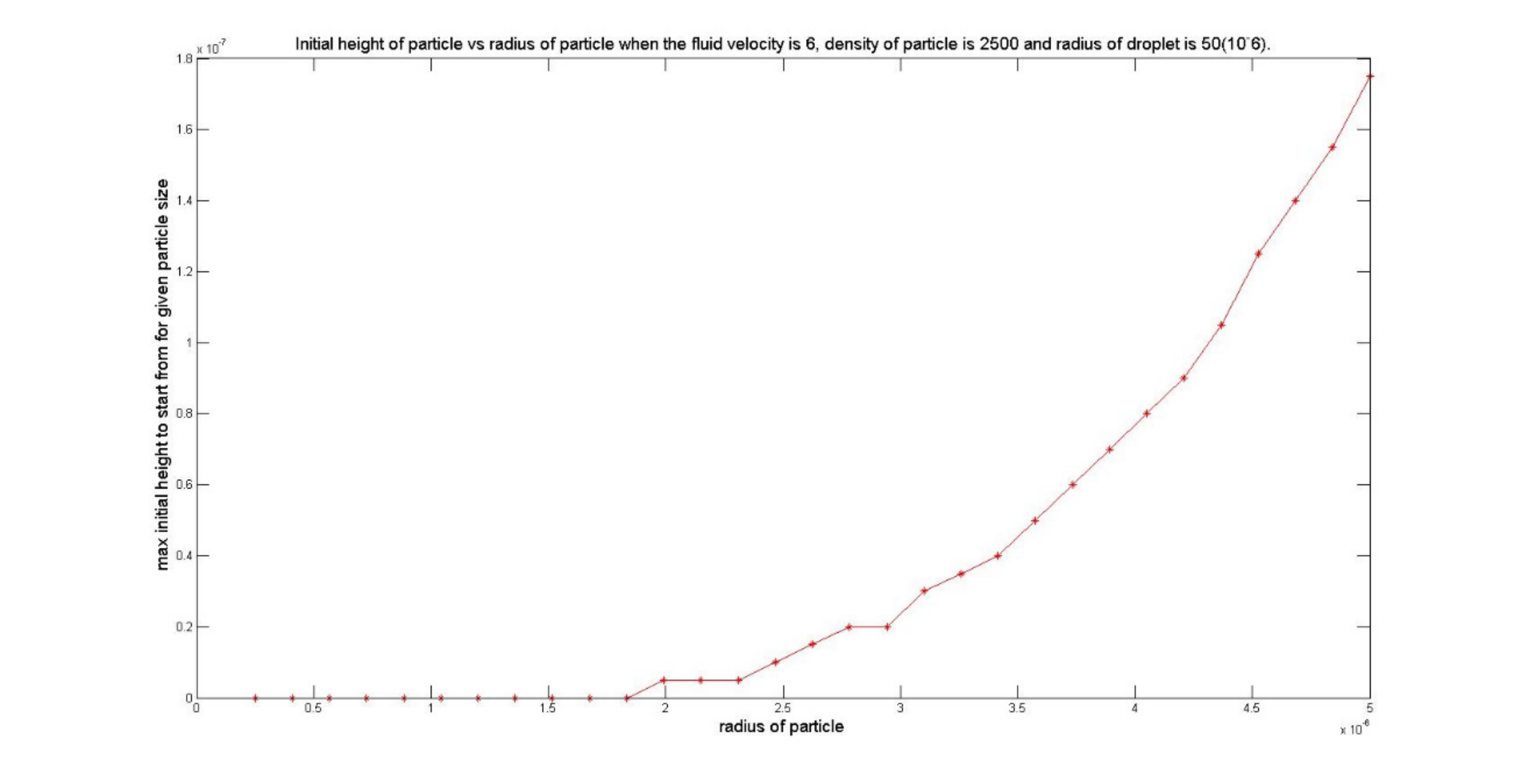
# Particles Colliding with Droplet



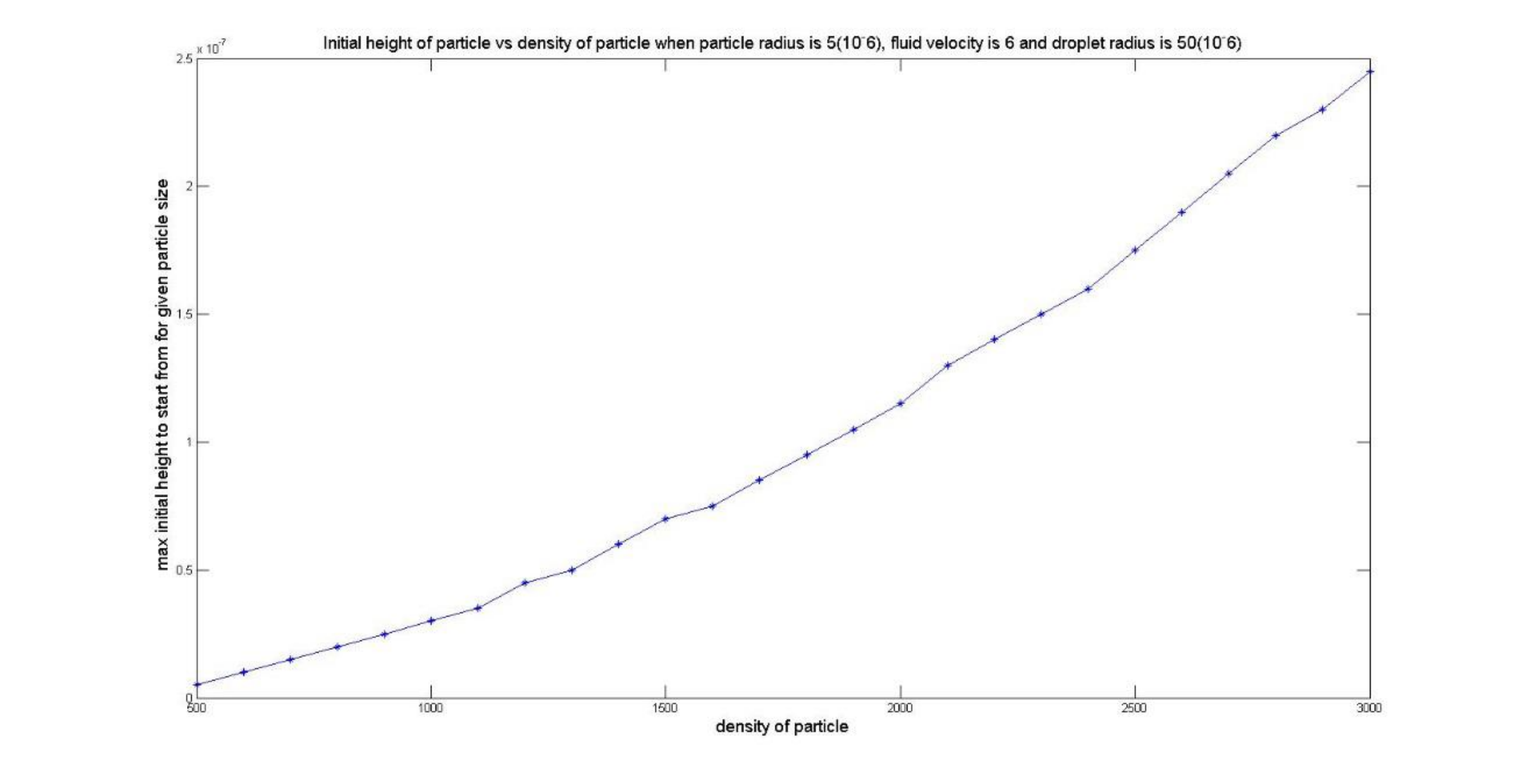
# Discussion of preliminary results

- Variation of 4 primary variables
- Calculation of maximum initial height to ensure collision
- Plot of maximum initial height vs. variable
- Determine dependencies

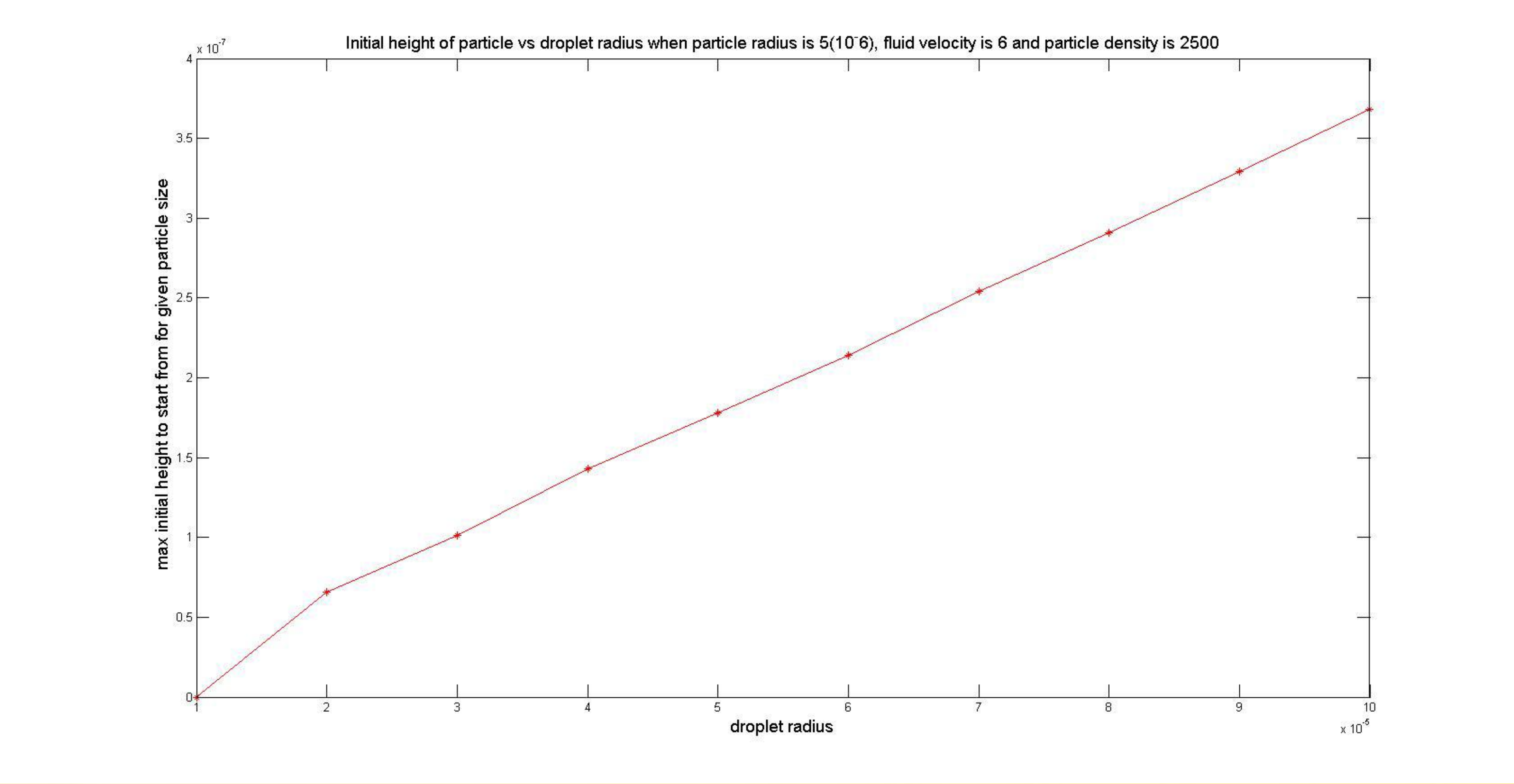
# Max. initial height with respect to particle size



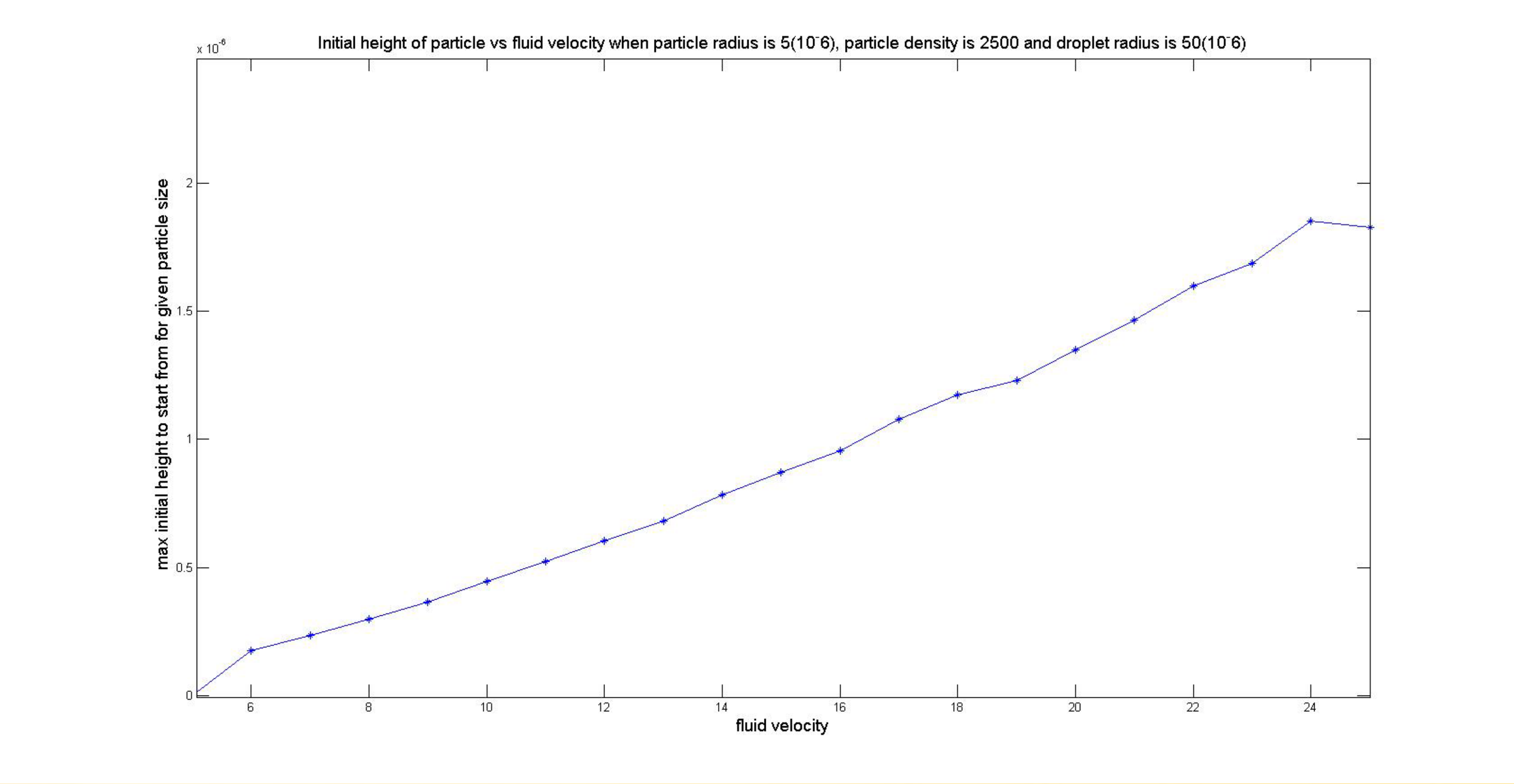
# Max. initial height with respect to density



# Max. initial height with respect to droplet size



# Max. initial height with respect to fluid velocity



# Future Work

- Use of higher order Runge-Kutta method to improve accuracy
- Improve accuracy of drag force calculation with Oseen‘s approximation
- Find a mathematical description of all parameters with respect to particle-droplet collisions
- Use information regarding dust concentration and water flow rates to determine the dust capture efficiency of a water spray system.
- Make a user friendly interface

# Conclusion

- Successfully completed first step in modeling dust particle and water droplet interaction for dust control
- Model behaves as logically expected
- Next steps can be implemented

# Glück Auf!